Plasma waves driven by gravitational waves in an expanding universe

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Abstract. In a Friedmann-Robertson-Walker (FRW) cosmological model with zero spatial curvature, we consider the interaction of the gravitational waves with the plasma in the presence of a weak magnetic field. Using the relativistic hydromagnetic equations it is verified that large amplitude magnetosonic waves are excited, assuming that both, the gravitational field and the weak magnetic field do not break the homogeneity and isotropy of the considered FRW spacetime.

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1. Introduction

In recent years, considerable progress has been made in understanding plasma phenomena in the radiation era (plasma epoch), a period of the expanding universe which has particular interest. It is known that after the end of the inflation, the universe enters standard Friedman-Robertson-Walker(FRW) stage and the wavelength of the large-scale perturbations increases less rapidly than the Hubble length which, at that time corresponds to the particle horizon (Zimdahl W 1997). It is known that at the end of the inflationary era density perturbations and gravitational waves(GW) are produced (Brevik I and Sandvik H B 2000, Lyth D H 1996). The density perturbations are thought to be responsible for the formation of large-scale structure along with a possible GW contribution. Moreover, the density perturbations due to the inflation are responsible for the observed anisotropies of the Cosmic Microwave Background Radiation(CMBR) (Kandaswamy Subramanian and Barrow J D 1998, Koh S and Lee C H 2000). On the other hand, it is argued (see Dimopoulos K et al 2001) that at the end of the inflationary epoch large-scale magnetic fields and inhomogeneities could be generated. The existence of large-scale magnetic fields in the early universe may have important consequences on the large scale structure formation process as shown in (Maartens R et all C 2001, Tsagas G C 2001, Grasso D and Rubinstein H R 2001, Tsagas G C and Maartens R 2000). The history of the magnetic fields from magnetogenesis in the early universe to the present time with the evolution and damping of the magnetic fields up to recombination era, is presented in (Jedamzik K et al 2000, Olinto V 1998). All this knowledge related to the density perturbations and magnetic inhomogeneities in the early stages of the universe, help us to construct possible scenarios to discuss plasma phenomena (see Holcomb K A and Tajima T 1989, Holcomb K A 1990, Dettmann et al 1993, Sahni V 1989, Ford L H and Parker L 1977, Caprini C and Durrer R 2001, Brodin G, Marklund M 1999, Marklund M et all 2000, Brodin G et al 2001, Brodin G et al 2001). In particular, from $t = 10^{-2}s$ to the time of recombination the ordinary matter content will be in the form of plasma. From approximately $t = 10^{-2}s$ to $t = 1s$, the plasma is dominated by electrons and positrons while from $t = 1s$ to the time of recombination at roughly $t = 10^{13}s$, the plasma consists mostly of electrons and protons with a admixture of ions of light elements. These plasmas are believed to be in thermal equilibrium, or at last strongly coupled, with photons. It is the photons however, which dominates the energy density of the universe until the time $\approx 10^{2}s$ Thus most of the period prior to the recombination is traditionally called the "radiation era" of the universe. In this paper we will focus on the radiation epoch of the universe and try to discuss, whether the existence of the GW in a FRW space time with zero spatial curvature and small magnetic field may produce instabilities. These

kind of problems are of particular interest since astrophysicists have began to discuss finite-amplitude wave propagation in hydromagnetic systems in the presence of GW (Papadopoulos D et al 2001, Papadopoulos D Vlahos L and Esposito F P 2001). Long ago Papadopolos and Esposito (Papadopoulos D Esposito F P 1982), derived the exact equations governing finite amplitude wave propagation in hydromagnetic media within the general theory of relativity, in the so called Cowling approximation (Cowling T G 1941). These equations are of particular interest because when they are linearized, they are very useful to discuss stability criteria in the presence of magnetic fields. Recently, (Papadopoulos et al. 2001) have derived a dispersion relation in an anisotropic cosmological model (Thorne K S 1967). They verified that in the case that the parameter γ, used to parametrize the equation of state, approaches 1/3, the anisotropic cosmological model becomes isotropic, actually a FRW with a scale factor $S(t) = t^{1/2}$, and the expansion θ of the universe prevents any instability. But in ((Dimopoulos K et al 2001) the authors argue that in FRW cosmological model, if one starts with small fluctuations, may obtain growth in those instabilities. The classical theory of GW perturbations on FRW universe has been worked out by Lifshitz and discussed by several authors (Ya.B Zel'dovich and Novikov I D 1983). In this paper we consider a FRW cosmological model,with signature $(-, +, +, +)$ witch is spatial flat and a weak magnetic field, perturbed by a GW with two polarization modes. As state by Marklund M et al 2000, the presence of a weak magnetic field propagating on the background FRW model couple to the gravitational wave producing a pulse of gravitationally induced electromagnetic wave. In principle, this breaks the homogeneity and isotropy of the FRW spacetime. However, it can be shown that a weak magnetic field is consistent with such a background provided that the spatial variations it generates are negligible. Therefore, we assume that both, the weak magnetic field and the GW, do not break the homogeneity and isotropy of the FRW space time. Applying the formalism presented in (Ya.B Zel'dovich and Novikov I D 1983, Brandenburg A et al 2000, Weinberg S 1972) in our scenario, we verify that density fluctuations may be obtained.

2. The Equations

The exact equations governing finite-amplitude wave propagation in hydromagnetic media in the frame of general relativity have been discussed in (Papadopoulos D Esposito F P 1982, Papadopoulos D. et al.2001, Papadopoulos D Vlahos L and Esposito F P 2001). For completeness we recast the relevant equations. We start with the Einstein field equations

$$R_{ab} - \frac{1}{2}g_{ab}R = -\kappa T_{ab}, \tag{1}$$

with

$$T^{ab}_{;b} = 0 \tag{2}$$

For simplicity we obtain $c = 1$ and $\kappa = \frac{8\pi G}{c^4} = 1$. Taking the covariant divergence of the Bianchi identities we obtain

$$(R^{ab} - \frac{1}{2}Rg^{ab})_{;ab} = 0 \tag{3}$$

where $R_{ab} = R^c_{abc}$ is the Ricci tensor and R_{abcd} is the curvature tensor defined by

$$u_{a;bc} - u_{a;cd} = u_d R^d_{abc} \tag{4}$$

For a unit time-like vector we choose $u_a u^a = -1$ and our hydromagnetic system will be specified by the following choice for the energy-momentum tensor

$$T^{ab} = (\epsilon + \frac{H^2}{2})u^a u^b + (p + \frac{H^2}{2})h^{ab} - H^a H^b \tag{5}$$

with

$$h^{ab} = g^{ab} + u^a u^b, \quad \epsilon = \rho + \rho\Pi \tag{6}$$

where u^a is the fluid velocity, ρ the mass density, $\rho\Pi$ the internal energy density, p the pressure of the fluid and H^a is the prevailing magnetic field as measured by an observer co-moving with u^a. Furthermore we introduce the expansion $\theta = u^a_{;a}$, the shear $\sigma_{ab} = h^c_a h^d_b u_{(c;d)} - \frac{1}{3}\theta h_{ab}$ and the twist $\omega_{ab} = h^c_a h^d_b u_{[c;d]}$, where the round bracket denotes symmetrization while the square bracket antisymmetrization. From Eq.(1) and Eq.(5) we find

$$R^{ab} = -\{\frac{1}{2}(\epsilon + 3p + H^2)u^a u^b + \frac{1}{2}(\epsilon - p + H^2)h^{ab} - H^a H^b\} \tag{7}$$

Substituting Eq.(7) into Eq.(3) we have

$$\ddot{x} + g^{ab}\left(p + \frac{H^2}{2}\right)_{;ab} + 2\dot{x}\theta + x_{;a}\dot{u}^a + x(\dot{\theta} + \theta^2 + \dot{u}^a_{;a}) - (H^a H^b)_{;ab} = 0 \tag{8}$$

where $\dot{u}^a = u^a_{;c}u^c$ and $x = \epsilon + p + H^2$ (Notice,that Eq.(8) can be obtained directly from Eq.(2) and Eq.(5)). Raychaudhuri's equation in the form

$$\dot{u}^a_{;a} = \dot{\theta} + \frac{\theta^2}{3} + 2(\sigma^2 - \omega^2) + \frac{1}{2}(\epsilon + 3p + H^2) \tag{9}$$

where $2\sigma^2 = \sigma^{ab}\sigma_{ab}$, $2\omega^2 = \omega^{ab}\omega_{ab}$ and dot means covariant derivative along u^a allows us to write the Eq.(8) as

$$\begin{aligned} &\ddot{x} + g^{ab}\left(p + \frac{H^2}{2}\right)_{;ab} + 2\dot{x}\theta + 2x\dot{\theta} + x_{;a}\dot{u}^a \\ &+2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2) + \frac{1}{2}x(\epsilon + 3p + H^2) - (H^a H^b)_{;ab} = 0 \end{aligned} \tag{10}$$

In order to make further progress with Eq.(10) we must use the equations of motion for the fluid and Maxwell's equations. These equations are

$$0 = T^{ab}_{;b} = \dot{x}u^a + x\dot{u}^a + x\theta u^a + (p + \frac{H^2}{2})_{;b}g^{ab} - (H^a H^b)_{;b} \tag{11}$$

and

$$\dot{H}^a = (\sigma^a_b + \omega^a_b - \frac{2}{3}\delta^a_b\theta)H^b + \frac{1}{\epsilon + p}p_{;b}H^b u^a \tag{12}$$

The last equation (12) may be written in the following form

$$\frac{\mu \dot{H^2}}{8\pi} = \frac{\mu}{4\pi}\sigma_{ij}H^i H^j - \frac{4\theta}{3}(\frac{\mu H^2}{8\pi}) \tag{13}$$

where μ is the permeability. Equations (11) and (12) imply that

$$\ddot{\epsilon} + (\dot{\epsilon} + \dot{p})\theta + (\epsilon + p)\dot{\theta} = 0 \tag{14}$$

Recalling that $x = \epsilon + p + H^2$ we find that Eq.(10) takes the form

$$\begin{aligned} &(\epsilon - \frac{H^2}{2})_{;ab}u^a u^b = h^{ab}(p + \frac{H^2}{2})_{;ab} + 2(\dot{H^2}\Theta) - (H^a H^b)_{;ab} \\ &+2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2 - \dot{u}^a \dot{u}_a) + \frac{x}{2}(\epsilon + 3p + H^2) \\ &+2\dot{u}_a(H^a H^b)_{;b} + (H^2)_{;a}\dot{u}^a \end{aligned} \tag{15}$$

The Eqs. (11),(12) and (15) may be applied to the investigation of perturbation effects and hence to the study of the linearizied stability criteria.

3. The Perturbed Equations

Following the well established method by (Hawking 1966), used also in (Papadopoulos D Esposito F P 1982, Papadopoulos D et al. 2001, Papadopoulos D Vlahos L and Esposito F P 2001), we consider the perturbations as follows:

$$\epsilon = \epsilon_0 + \delta\epsilon, p = p_0 + \delta p \tag{16}$$

$$u^\mu = u^\mu_0 + \delta u^\mu, H^\mu = H^\mu_0 + \delta H^\mu \tag{17}$$

where the index 0 means unperturbed quantities.

Taking into account the Eqs.(16),(17), we perturbed the equations Eqs.(15),(11) and (12) respectively, keeping only linear terms in the perturbed equations and assuming that $\delta g_{ab} \neq 0$. Thus we have

$$\begin{aligned} &\delta[(\epsilon - \frac{H^2}{2})_{;ab}]u^a u^b - \delta\Gamma^k_{ab}(\epsilon - \frac{H^2}{2})_{,k}u^a u^b \\ &+ (\epsilon - \frac{H^2}{2})_{;ab}(u^a\delta u^b + u^b\delta u^a) + (\delta\epsilon - H^c\delta H_c)_{;ab}u^a u^b \end{aligned}$$

$$
\begin{aligned}
&= \delta g^{ab} p^*_{,ab} + (u^a \delta u^b + u^b \delta u^a) p^*_{ab} + h^{ab}(\delta p^*_{,ab} - \delta\Gamma^k_{ab} p^*_{,k}) \\
&+ 2\delta[(H^2)_{,c}\Theta + H^2 \Theta_{,c}]u^c - H^b[2\delta(H^a_{,ab}) + H^k \delta(\Gamma^a_{ak,b}) + 2\delta\Gamma^a_{ak} H^k_{,b}] \\
&- 2H^a_{,ab}\delta H^b - 2\delta\Gamma^a_{al} H^l H^b_{,b} - \delta(H^a_{,b}) H^b_{,a} - H^a_{,b}\delta(H^b_{,a} \\
&- \delta\Gamma^a_{bl} H^l H^b_{,a} - H^a H^c \delta\Gamma^b_{ac,b} - \delta\Gamma^b_{ac} H^c_{,b} H^a + 2x[\sigma\delta\sigma - u^c u^d(\delta u^a_{,c} u_{a,d} + u^a_{,c}\delta u_{a,d} \\
&+ \delta\Gamma^a_{cl} u^l u_{a,d} - u^a_{,c} u_m \delta\Gamma^m_{ad}) - (u^d \delta u^c + u^c \delta u^d) u^a_{,c} u_{a,d}] \\
&+ \frac{1}{2}(\epsilon + 3p + H^2)(\delta\epsilon + \delta p + \delta H^2) + \frac{1}{2}x(\delta\epsilon + 3\delta p + \delta H^2) \\
&+ [2\delta u^c u_{a,c} + 2u^c(\delta u_{a,c} - \delta\Gamma^m_{ac} u_m)](H^a \delta H_b + H^b \delta H^a) + (H^2)_{;a}(\delta u^a)_{;c} u^c \\
&+ (H^2)_{,a} u^a_{,c}\delta u^c + (H^2)_{,a} u^c(\delta u^a_{,c} + \delta\Gamma^a_{cm} u_m)
\end{aligned}
\tag{18}
$$

The perturbed Maxwell's equations are

$$
\delta H^b_{,0} + \delta\Gamma^b_{0l} H^l = -\delta\sigma^b_a H^a + \frac{2}{3} H^b \delta\theta - \frac{u^b}{\epsilon + p}\delta p_{,c} H^c \tag{19}
$$

and the perturbed Eqs.(11) give

$$
\begin{aligned}
-(\epsilon + p + H^2)\delta u^c_{,0} = h^c_a[&H^b \delta H^a_{,b} + \delta\Gamma^a_{bl} H^b H^l \\
&+ H^a \delta H^b_{,b} + \delta\Gamma^b_{bm} H^m H^a] - \delta h^{cb} p^*_{,b} - h^{cb}\delta p^*_{,b}
\end{aligned}
\tag{20}
$$

We will apply these equations to a spacial flat FRW space-time given by the equation (see Ya.B Zel'dovich and Novikov I D 1983)

$$
ds^2 = -c^2 dt^2 + S^2[(1 + \frac{h_1}{S^2})dx^2 + (1 - \frac{h_1}{S^2})d^2y + d^2z] + 2h_2 dxdy \tag{21}
$$

where $S = S(t)$ is the scale factor of the FRW universe, parametrazing the universe expansion with $h_1 = h_+(t,z), h_2 = h_\times(t,z)$ are the two components of the GW corresponding to the two possible polarizations. For the metric (21) the non-zero Γ^c_{ab} are

$$
\begin{aligned}
&\Gamma^0_{11} = SS_{,0} + \frac{1}{2}h_{1.0}, \Gamma^0_{12} = \frac{1}{2}h_{2,0}, \\
&\Gamma^0_{22} = SS_{,0} - \frac{1}{2}A_{1,0}, \Gamma^0_{33} = SS_{,0}, \\
&\Gamma^1_{01} = \frac{S_{,0}}{S} + \frac{1}{2}(\frac{h_{1,0}}{S^2} - 2\frac{h_1 S_{,0}}{S^3}), \\
&\Gamma^1_{02} = +\frac{1}{2}(\frac{h_{2,0}}{S^2} - 2\frac{h_2 S_{,0}}{S^3}), \\
&\Gamma^1_{13} = \frac{1}{2S^2}h_{1,z0}, \Gamma^1_{23} = \frac{1}{2S^2}h_{2,z0}, \Gamma^2_{01} = +\frac{1}{2}(\frac{h_{2,0}}{S^2} - 2\frac{h_2 S_{,0}}{S^3}), \\
&\Gamma^2_{02} = \frac{S_{,0}}{S} - \frac{1}{2}(\frac{h_{1,0}}{S^2} - 2\frac{h_1 S_{,0}}{S^3}), \Gamma^2_{13} = \frac{1}{2}\frac{h_{2,z}}{S^2}, \Gamma^2_{23} = -\frac{1}{2S^2}h_{1,z}, \\
&\Gamma^3_{03} = \frac{S_{,0}}{S}, \Gamma^3_{11} = -\frac{1}{2S^2}h_{1,z}, \Gamma^3_{12} = -\frac{1}{2S^2}h_{2,z}, \Gamma^3_{22} = \frac{1}{2S^2}h_{1,z}
\end{aligned}
\tag{22}
$$

In an expanding universe described by the metric (21), we obtain $S(t) = t^{1/2}$ and consider an equation of state $p = \frac{1}{3}\epsilon$(radiation era). We assume that $u^a = (-1, 0, 0, 0)$, $H^a = \frac{1}{S^3}(0, H_0^1, H_0^2, H_0^3)$ (Holcomb 1990), with $H_0^a = const, H_0^2 = (H_0^1)^2 + (H_0^2)^2 + (H_0^3)^2$, $\delta H^a = \frac{1}{S^2}(0, \delta H^1, \delta H^2, \delta H^3)$, $\delta u^a = (\delta u^0, \delta u^1, \delta u^2, \delta u^3)$ and $k^a = (0, 0, 0, k)$ where k is the magnitude of the comoving wavelength (momentum). Thus, the perturbed Maxwell's equations (19) take the form

$$
\begin{aligned}
&\delta H^1_{,0} = \theta\delta H^1 + H^1\delta\theta - \frac{1}{3}(\theta H^1 - H^1_{,0}))\delta u^0 \\
&-\frac{1}{2}H^3\delta u^1_{,3} - \frac{\theta}{6S^2}(H^1 h_1), \\
&\delta H^2_{,0} = \theta\delta H^2 - \frac{1}{2}H^3\delta u^2_{,3} - \frac{\theta}{6S^2}(H^1 h_2), \\
&\delta H^3_{,0} = \theta\delta H^3 + H^3\delta\theta - \frac{1}{3}\theta H^3\delta u^0 - \frac{1}{2}H^1\delta u^1_{,3} - H^3\delta u^3_{,3}
\end{aligned}
\tag{23}
$$

and the equations of motion (20), reduce to the equations

$$
\begin{aligned}
&\delta u^0_{,0} = 0 \\
&x\delta u^1_{,0} = \frac{2}{3}x\theta\delta u^1 - H^3\delta H^1_{,3} - H^1\delta H^3_{,3} - \frac{(H^1H^3)}{S^2}h_{1,3}, \\
&x\delta u^2_{,0} = \frac{2}{3}x\theta\delta u^2 - H^3\delta H^2_{,3} - \frac{(H^1H^3)}{S^2}h_{2,3}, \\
&x\delta u^3_{,0} = \frac{2}{3}x\theta\delta u^3 - 2H^3\delta H^3_{,3} + \frac{1}{S^2}H_{0\mu}\delta H^\mu_{,3} + \delta p_{,3} + (H^1)^2 h_{1,3}
\end{aligned}
\tag{24}
$$

In this case the Eq.(18) becomes:

$$
\begin{aligned}
&\delta\epsilon_{,00} - \frac{1}{S^2}\delta p_{,33} - \frac{1}{S^3}H_{0\mu}\delta H^\mu_{,00} - \frac{1}{S^5}H_{0\mu}\delta H^\mu_{,33} + \frac{2H_0^3}{S^3}\delta H^3_{,33} \\
&-\theta\delta p_{,0} + \frac{\theta}{3}\frac{H_{0\mu}\delta H^\mu_{,0}}{S^3} - [\frac{5\theta^2}{18} + 2(\epsilon + 2p + \frac{H_0^2}{S^4})]\frac{(H_{0\mu}\delta H^\mu}{S^3}) \\
&-4\theta\delta\theta(\epsilon + p - \frac{H_0^2}{3S^2}) + \frac{H_0^2}{S^4}\delta\theta_{,0} - \frac{4\theta^2}{3}(\delta\epsilon + \delta p) \\
&-\frac{1}{2}(\delta\epsilon + \delta p)(\epsilon + 3p) - \frac{1}{2}(\delta\epsilon + 3\delta p)(\epsilon + p) - \frac{H_0^2}{S^4}(\delta\epsilon + 3\delta p) \\
&= (H^1)^2 h_1[-\frac{k_g^2}{S^2} + \epsilon + 2p + \frac{H_0^2}{S^4} - \frac{\theta^2}{3}]
\end{aligned}
\tag{25}
$$

It is known that in an expanding universe the perturbations can not be solutions for the GW, because the amplitude of such a wave must decay in time and its frequency must redshift. Thus, writing the wave equation for a free wave in a cosmological background and requiring a propagating solution, the spatial part has the usual form $\exp(ikz)$, while the time depended part has the form $\exp(i\omega t/S)$. In the plasma period, we assume that the perturbations, Eqs.(16),(17), have the form (Ya.B Zel'dovich and Novikov I D 1983): $\delta H^a = \frac{1}{S^2}\exp(ikz - i\omega t)$, $\delta\epsilon = \frac{1}{S^4}\exp(ikz - i\omega t)$,

$\delta u^a = \delta u_0^a \exp{(ikz - i\omega t)}$ and the two components of the GW (Brandenburg A et al 2000, Weinberg S 1972) $h_1, h_2 = \frac{1}{S}\exp{(ik_g z - i\omega_g t)}$ where $\omega = \frac{\omega_i}{S}$, $\omega_g = \frac{\omega_{gi}}{S}$ and ω_i, ω_{gi} are the frequencies at the initial time $t_i < t$. Under these considerations, Eq.(25) yields a general dispersion relation for the coupling of GW with MHD waves which is:

$$\Lambda R_1 \delta\epsilon + \frac{1}{S^3} H_{0a} \delta H^a R_2 = h_1 \Lambda (H^1)^2 R_3 \tag{26}$$

where

$$\begin{aligned}
&\Lambda = \frac{i\omega}{2}(1 - \frac{2u_A^2}{S^4}) + \frac{\theta}{3}(1 + \frac{4u_A^2}{S^4}) \\
&R_1 = -\frac{\omega^2}{4}(1 + \frac{u_A^2}{S^4}) + \frac{i\omega}{2}\theta(1 + 5c_s^2 - 10\frac{u_A^2}{S^4}) + \frac{k^2 c_s^2}{S^2} \\
&-\frac{1}{2}(\epsilon + 3p)(1 + c_s^2) - \frac{1}{2}(\epsilon + p)(1 + 3c_s^2) - \frac{H_0^2}{S^4}(1 + 3c_s^2) \\
&R_2 = \frac{\omega^2}{4}(1 - \frac{u_A^2}{S^4}) - \frac{8i\omega u_A^2}{2S^4} + \frac{k^2}{S^2} - 2(\epsilon + 2p + \frac{H_0^2}{S^4}) + \theta^2(\frac{28}{9}\frac{u_A^2}{S^4} - \frac{47}{3}) \\
&R_3 = -\frac{k_g^2}{S^2} + \epsilon + 2p + \frac{H_0^2}{S^4} - \frac{5i\theta\omega_g}{2} + 8\theta^2 \\
&-\frac{u_A^2}{S^4}[\frac{\omega_g^2}{4} - \frac{9i\theta\omega_g}{6S} + \frac{16}{3}\theta^2]
\end{aligned} \tag{27}$$

and $u_A^2 = \frac{\upsilon_a^2}{1+\upsilon_A^2}$, $\upsilon_A^2 = \frac{H^2}{\epsilon}$ is the Alfven speed and c_s is the sound speed. It is important to emphasize that, the second component of GW does not appear in Eq.(26). Besides, the right hand side of Eq.(26) is proportional of H^1. This suggests that the dispersion relation (26) may be used for waves that correspond to the magnetosonic mode $(H^1 \neq 0, H^3 = 0)$.

4. The dispersion relation

Furthermore, obtaining $H^1 \neq 0$ and $H^3 = 0$, the Maxwell's equations and the equations of motion give the following results respectively: $\delta H^3 = \delta H^2 = 0$,

$$\Lambda H_1 \delta H^1 = -\frac{u_A^2}{S^4}(\frac{\theta}{3} - \frac{i\omega}{2})(1 + c_s^2)\delta\epsilon + h_1 \frac{H_0^2}{S^6}[-\frac{u_A^2}{S^4}(\frac{4\theta}{3} - \frac{i\omega_g}{2}) + \frac{\theta}{3}] \tag{28}$$

and $\delta u^1 = 0$,$\delta u^2 = 0$. We notice that in all equations above equations we have taken $c = 1$, $\kappa = 8\pi G/c^4 = 1$ and we have put $H = B \rightarrow \sqrt{4\pi}H$ (from Gaussian units), where κ is the coupling constant in Einstein's field equations. Substituting Eq.(28) into Eq.(26) and using ordinary units, we obtain

$$(A_r + iA_i)\delta\epsilon = h_1 \frac{(H^1)^2}{4\pi}(B_r + iB_i) \tag{29}$$

where

$$
\begin{aligned}
&A_r = -(\frac{\omega}{2})^2\Xi_2 + \Xi_0, A_i = -(\frac{\omega}{2})^3\Xi_3 + (\frac{\omega}{2})\Xi_1,\\
&B_r = -(\frac{\omega}{2})^2\Pi_2 + \frac{\omega\omega_g}{4}\Pi_1 + \Pi_0,\\
&B_i = -(\frac{\omega}{2})^2 M_2 + (\frac{\omega}{2})nM_1 + M_0
\end{aligned} \tag{30}
$$

with

$$
\begin{aligned}
&\Xi_3 = (1+\frac{c^2u_A^2}{S^4})(1-\frac{2u_A^2}{c^2S^4}) + \frac{u_A^2}{c^2S^4}(1+\frac{c_s^2}{c^2})(1-\frac{u_A^2}{c^2S^4})\\
&\Xi_2 = \theta[(1-\frac{2u_A^2}{c^2S^4})(1+5\frac{c_s^2}{c^2}-10\frac{u_A^2}{c^2S^4}) - \frac{1}{3}(1-25\frac{u_A^2}{c^2S^4}(1+\frac{c_s^2}{c^2})]\\
&\Xi_1 = (1-\frac{2u_A^2}{c^2S^4})[\frac{k^2c_s^2}{S^2} - \frac{8\pi G}{c^4}\frac{1}{2}(\epsilon+3p)(1+\frac{c_s^2}{c^2})\\
&-\frac{4\pi G}{c^4}(\epsilon+p)(1+3\frac{c_s^2}{c^2}) - \frac{2G}{c^4}\frac{H_0^2}{S^4}(1+3\frac{c_s^2}{c^2})]\\
&+\frac{u_A^2}{c^2S^4}(1+\frac{c_s^2}{c^2})[\frac{k^2c^2}{S^2} - \frac{16\theta\pi G}{3c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4})(1+\frac{c_s^2}{c^2})(1-25\frac{u_A^2}{c^2S^4})]\\
&\Xi_0 = \frac{\theta}{3}\{(1+\frac{4u_A^2}{c^2S^4})[\frac{k^2c_s^2}{S^2} - \frac{4\pi G}{c^4}(\epsilon+3p)(1+\frac{c_s^2}{c^2})\\
&-\frac{4\pi G}{c^4}(\epsilon+p)(1+3\frac{c_s^2}{c^2}) - \frac{2G}{c^4}\frac{H_0^2}{S^4}(1+3\frac{c_s^2}{c^2})]\\
&-\frac{u_A^2}{c^2S^4}(1+\frac{c_s^2}{c^2})[\frac{k^2c^2}{S^2} - \frac{16\pi G}{c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4}) + \theta^2(\frac{28u_A^2}{9c^2S^4} - \frac{47}{3})]\}
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
&\Pi_2 = \frac{\theta}{3}(1-\frac{u_A^2}{c^2S^4})(1-\frac{4u_A^2}{c^2S^4})\\
&\Pi_1 = 5\theta[(1-\frac{2u_A^2}{c^2S^4})(1-\frac{3}{5}\frac{u_A^2}{c^2S^4}) - \frac{8}{5}\frac{u_A^2}{c^2S^4}]\\
&\Pi_0 = \frac{\theta}{3}\{(1+\frac{4u_A^2}{c^2S^4})[-\frac{k_g^2c^2}{S^2} - \frac{u_A^2\omega_g^2}{4c^2S^4} + \frac{8\pi G}{c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4})\\
&+8\theta^2(1-\frac{2u_A^2}{3c^2S^4})] + (1-\frac{4u_A^2}{c^2S^4})[\frac{k^2c^2}{S^2}\\
&-\frac{16\pi G}{c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4}) + \theta^2(\frac{28u_A^2}{9c^2S^4} - \frac{47}{3})]\}
\end{aligned} \tag{32}
$$

and

$$
\begin{aligned}
&M_2 = \frac{u_A^2\omega_g}{2c^2S^4}(1-\frac{u_A^2}{c^2S^4})\\
&M_1 = (1-\frac{2u_A^2}{c^2S^4}[-\frac{k_g^2c^2}{S^2} - \frac{u_A^2\omega_g^2}{4c^2S^4} + \frac{8\pi G}{c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4})\\
&+8\theta^2\frac{u_A^2}{c^2S^4}(1-\frac{4u_A^2}{c^2S^4})]
\end{aligned}
$$

$$M_0 = -\frac{5\theta^2}{3}\frac{\omega_g}{2}(1-\frac{4u_A^2}{c^2S^4})(1-\frac{3u_A^2}{5c^2S^4})$$
$$+\frac{u_A^2\omega_g}{2c^2S^4}[\frac{k^2c^2}{S^2}-2\frac{8\pi G}{c^4}(\epsilon+2p+\frac{H_0^2}{4\pi S^4})+\theta^2(\frac{28u_A^2}{9c^2S^4}-\frac{47}{3})] \tag{33}$$

From now on, we will use the notation $n = \frac{\omega}{2}$ and $n_g = \frac{\omega_g}{2}$.

It is evident that this section 4, from the beginning, relies heavily on equations and calculations, therefore we shall try to present our case by splitting this section in two subsections, one dealing with the gravitational-wave-free case and another one addressing the impact of the gravitational wave. Notice that the coefficient of the $\delta\epsilon$ on the left hand side of Eq.(29) describes the dispersion relation of the fluid. While, the coefficient of h_1 on the right hand side of Eq.(29) describes the dispersion relation of the gravitational wave.

4a. The gravitational-wave-free case

In the absence of the gravitational wave ($h_1 = 0$), the dispersion relation (Eq.(29)) is:

$$A_r + iA_i = 0 \Rightarrow (-n^2\Xi_2 + \Xi_0) + i(-n^3\Xi_3 + n\Xi_1) = 0 \tag{34}$$

Eq.(34) has three roots, two complex conjugates and one real; we examine one of them, say the $n_1 = n_r + in_i$, with $n_i << n_r$, since its complex conjugate will give the same information regarding the fluid and the real one is not important from the physical point of view. We substitute n_1 in Eq.(34) and the resulting equation after neglecting non-linear terms of n_i is:

$$-(n_r^2 + 2in_in_r)\Xi_2 + \Xi_0 - i(n_r^3 + 3in_in_r^2)\Xi_3 + i(n_r + in_i)\Xi_1 \simeq 0 \tag{35}$$

From the imaginary part of Eq.(35), we obtain $n_r^2 \simeq -2n_i\frac{\Xi_2}{\Xi_3} + \frac{\Xi_1}{\Xi_3}$, and $n_i \simeq \frac{\Xi_1\Xi_2 - \Xi_0\Xi_3}{2(\Xi_2^2 + \Xi_3\Xi_1)}$. After some elementary calculations, we verify that the condition $n_i << n_r$ is satisfied. Thus, from Eq.(29) in the absence of the GW, we obtain Eq.(34) and find the root n_1 which has a real part n_r, given by the equation

$$n_r^2 \simeq \frac{\frac{10\theta^2}{81}[\frac{17k^2c_s^2}{S^2} + \frac{2}{3}\frac{k^2u_A^2}{S^6}]}{[\frac{k^2c_s^2}{S^2} + \frac{2}{3}\frac{k^2u_A^2}{S^6} + \frac{400\theta^2}{81}]} + \frac{k^2c_s^2}{S^2} + \frac{2k^2u_A^2}{3S^6} \tag{36}$$

and an imaginary part, given by the equation

$$n_i \simeq \frac{\frac{\theta}{18}[\frac{17k^2c_s^2}{S^2} + \frac{2}{3}\frac{k^2u_A^2}{S^6}]}{[\frac{k^2c_s^2}{S^2} + \frac{2}{3}\frac{k^2u_A^2}{S^6} + \frac{400\theta^2}{81}]} \tag{37}$$

Note, that the two frequencies, n_r and n_i, depend on the expansion θ. If, somehow, the universe would stop to expand, then the real frequency n_r is just the so called magneto-sound frequency (Jedamzik K et all 2000, Olinto V 1998), while the frequency n_i becomes zero. In an expanding universe, an imaginary part of the frequency n_i appear, and the real frequency n_r, is shifted by a term proportional to θ^2. The

existence of the frequency n_r means that before the GW starts to interact with the plasma, the plasma were oscillating with this n_r normal frequency, while the frequency n_i corresponds to just a negligible noise.

4b. The dispersion relation in the presence of the gravitational wave.

Now we consider the case where the GW interacts with plasma. We assume that the GW is weak and does not affect the background metric and the frequencies n_r and n_i. In this scenario, we write: $\delta\epsilon \simeq \frac{1}{S^4}\delta\epsilon_0 \exp i(nt - kz)$, and $h_1 \simeq \frac{1}{S}h_{10} \exp i(n_g t - k_g z)$ (Brandenburg A et all 2000, Weinberg S 1972), where $\delta\epsilon_0 = constant$ and $h_{10} = constant$. Thus, Eq.(29) is written as

$$\delta\epsilon_0 \simeq h_{10} e^{i[(n-n_g)t-(k_g-k)z]} \frac{H_0^2}{S^3}[\frac{B_r + iB_i}{A_r + iA_i}] \tag{38}$$

The term $T = \frac{B_r+iB_i}{A_r+iA_i}$ takes its maximum value close to the frequency $n = n_1$. Let n_g and k_g, the frequency and the wave number of the GW respectively. Close to the frequency n_1, we assume that the frequency of the driving GW n_g coincides with the frequency $n_r = Re(n_1)$. The produced magnetosotic wave has $n_r = n_g = k_g c$ and the two wave numbers differ by an amount Δk, e.g. $k = k_g + \Delta k$. Taking into account all the above considerations, Eq.(38) reads

$$\delta\epsilon_0 \approx h_{10} e^{-iz\Delta k} e^{n_i t} \frac{H_0^2}{4\pi S^3}[\frac{B_g}{\Delta k A_k} + \frac{B_k}{A_k}] \tag{39}$$

In the linear theory of perturbations we have $\delta\epsilon < \epsilon_0$. Thus, Eq.(39) gives

$$h_{10} e^{-iz\Delta k} e^{n_i t} \frac{H_0^2}{4\pi S^3}[\frac{B_g}{\Delta k A_k}] < \epsilon_0 = const., \ as \ \ \Delta k \to 0. \tag{40}$$

where

$$B_g = \frac{\theta}{3}(14n_g^2 - 8\frac{k_g^2 u_A^2}{S^6}) - n_i(-\frac{k_g^2 c^2}{S^2} + \frac{k_g^2 u_A^2}{S^6})$$
$$+ i[\frac{13\theta}{3} n_i n_g + n_g(-\frac{k_g^2 c^2}{S^2} + \frac{3k_g^2 u_A^2}{S^6} - \frac{5\theta^2}{3})] \tag{41}$$

$$B_k = \frac{\theta}{3}[\frac{2k_g c^2}{S^2} - \frac{4k_g u_A^2}{S^6}] + in_g(\frac{2k_g u_A^2}{S^6}) \tag{42}$$

and

$$A_k = \frac{\theta}{3}(\frac{2k_g c_s^2}{S^2}) - n_i(\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6}) + in_g(\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6}) \tag{43}$$

Because of the form of n_i (Eq.(37)) the quantity e^{tn_i} approaches to a constant value, as t approaches to very large values. Thus, to the approximations we made, the disturbance $\delta\epsilon_0$ does not exhibit growth. However, the term e^{tn_i} enhanced the amplitude of the oscillation and this may cause further enhancement on a non-linear approach which we will not discuss in this work. (The result is in agreement with the result obtained in (Holcomb K A and Tajima T 1989, Holcomb K A 1990). Alfven or

other MHD oscillations are not enhanced, but magnetosonic frequencies through the interaction with the GW become dominant. Therefore, it is interesting to know the maximum length which the GW will travel in the plasma and the maximum time the GW will interact with the plasma. From Eq.(40) we have

$$h_{10}e^{n_i t}\frac{H_0^2}{4\pi S^3}[\frac{Re(B_g A_k^*)}{A_k A_k^* \epsilon_0}] < \Delta k \tag{44}$$

Eq.(44) can give us approximately the Δk and subsequently, we can calculate the maximum distance L over which the two waves may interact coherently. That maximum L can be calculated in terms of the wavelength λ_g through the relation $L = \lambda_g(1 - \frac{\Delta k}{k_g})$ e.g

$$L < \lambda_g\{1 - h_{10}e^{n_i t}\frac{H_0^2}{k_g\epsilon_0}[\frac{I_1\frac{\theta}{3} + n_g^2 I_2}{I_3\frac{2\theta k_g c_s^2}{3S^2} + n_g^2 I_4}]\} \tag{45}$$

where

$$\begin{aligned} I_1 = {} & \frac{2\theta k_g c_s^2}{3S^2}(14n_g^2 - \frac{8k_g^2 u_A^2}{S^6}) - n_i[\frac{2k_g c_s^2}{S^2}(-\frac{k_g^2 c^2}{S^2} + \frac{k_g^2 u_A^2}{S^6}) \\ & + (14n_g^2 - \frac{8k_g^2 u_A^2}{S^6})(\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6})] \end{aligned} \tag{46}$$

$$I_2 = (\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6})(\frac{13\theta n_i}{3} - \frac{k_g^2 c^2}{S^2} + \frac{3k_g^2 u_A^2}{S^6} - \frac{5\theta^2}{3}) \tag{47}$$

$$I_3 = \frac{\theta}{3}\frac{2k_g c_s^2}{S^2} - 2n_i(\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6}) \tag{48}$$

and

$$I_4 = (\frac{2k_g c_s^2}{S^2} + \frac{4k_g u_A^2}{3S^6})^2 \tag{49}$$

The right hand side of Eq.(45) is smaller than the particle (Peacock J A 2000) horizon of comoving radius $r_H = c\int \frac{dt}{S}$. Hence, in a small fraction of the universe radius, there exists an interaction between the GW and the plasma producing small fluctuations with amplitude $P = h_{10}\exp i[n_i t - \Delta k z]\frac{H_0^2}{S^3}[\frac{B_r + iB_i}{A_r + iA_i}]$. During the interaction, energy of the order P^2L^3 is transferred from the GW to the fluid and magnetosonic waves are produced. These magnetosonic waves do not get damped since in our model we do not include dissipating processes. In a future paper we intend to study a more realistic model and investigate further astrophysical implications.

3.Discussion

In the present paper, we have applied the equations governing finite amplitude wave propagation in hydromagnetic media discussed by Papadopoulos and Esposito (Papadopoulos D Esposito F P 1982), in the so called Cowling approximation in a perturbed spatial flat FRW cosmological model. We have assumed that, despite the presence of the magnetic field and the perturbation h_1, the homogeneity and isotropy of the considered cosmological model do not change. Upon the consideration of those equations, we have derived a dispersion relation (Eq.(29)) in the radiation era (plasma epoch) . We found that, if the gravitational waves and the magnetic field (produced after inflation period) are parallel, then the gravitational waves may excite fluctuations which appear as fast magnetosonic waves. From Eq.(29), it is evident that both, the magnetic field and the gravitational wave should be different from zero otherwise we do not have genaration of density fluctuations. The amplitude of the magnetosonic wave is proportional to the gravitational wave and does not exhibit a damping since in our model we do not take into account dissipative processes. The process we suggest seems to be reversible in the sense that energy is transfered from the gravitational wave to the plasma exciting magnetosonic waves and vise versa. The transfer of this energy from one side to the other occurs without loses and is of the order P^2L^3. Also, we have found that in an expanding universe, in the absence of the gravitational wave, a new imaginary part of the frequency (n_i) appears and the real part is shifted by a term proportional to θ^2. In the presence of the gravitational wave, the interaction of the plasma with the gravitational wave produces magnetosonic waves oscillating at a frequency $n_r = n_g = k_g c$. Finally, the application of the above equations to the study of small disturbances is rather straightforward although tedious at this time, but we hope to be able to discuss, in a future publication, these equations in more realistic hydromagnetic media.

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